N-DOT: Networking by Duality, Optimization, and Topology

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Overview

Mathematical foundation
- Optimization
- Duality
- Topology

Network science and engineering
- Holistic analysis and structured design
- Protocol stack design
- Wireless communication networks

An emerging, alternative foundation for networking:
- Mathematically rich and practically relevant
- Promising track record of visible impacts
TCP solves the Basic NUM:

\[
\begin{align*}
\text{maximize} & \quad \sum_s U_s(x_s) \\
\text{subject to} & \quad \sum_{s : l \in L(s)} x_s \leq c_l, \quad \forall l, \\
x & \geq 0
\end{align*}
\]

Network as an (primal-dual distributed) optimization solver!
Can the number of equilibrium be:
0? No
1? Maybe, can check with sufficient conditions
2? Almost never
3? Maybe
∞? Almost never

Poincare-Hopf Index Theorem is the key!
Layering as Optimization Decomposition

Layering architecture: Decomposition scheme

Layers: Decomposed subproblems

Interface: Functions of primal or dual variables

\[
\begin{align*}
\text{maximize} & \quad \sum_i U_i(x_i, P_{e,i}) + \sum_j V_j(w_j) \\
\text{subject to} & \quad Rx \preceq c(w, P_e) \\
& \quad x \in C_1(P_e) \\
& \quad x \in C_2(F) \\
& \quad R \in R \\
& \quad F \in F \\
& \quad w \in \mathcal{W}
\end{align*}
\]
Systematically explore the space of alternative decompositions

Alternative Decompositions

Evolution of $\lambda^4$ for all methods

Iteration

Method 1 (subgradient)

Method 2 (Gauss−Seidel for all lambdas and gamma)

Method 3 (Gauss−Seidel for each lambda and gamma sequentially)

Method 4 (subgradient for gamma and exact for all inner lambdas)

Method 5 (subgradient for all lambdas and exact for inner gamma)

Method 6 (Gauss−Seidel for all lambdas and exact for inner gamma)

Method 7 (Jacobi for all lambdas and exact for inner gamma)
Duality: Convex Case

TCP congestion control

\[ x_s, \mu_s \rightarrow \text{Network} \]

\[ \lambda' = \sum_{l \in L(s)} \lambda_l \]

\[ R'^{l} = 1 - \sum_{l \in L(s)} E_l(r_l) \]

\[ \lambda_l, r_l \rightarrow \text{Link } l \]

\[ x'^l = \sum_{s \in S(l)} x_s \]

\[ \mu'^l = \sum_{s \in S(l)} \mu_s \]

Shannon’s channel capacity

\[
\begin{align*}
R(D) & \quad \text{Shannon} \\
C(S) & \quad \text{R(D)}
\end{align*}
\]

Dual R(D) Dual C(S)

Shannon and Lagrange duality

\[
\begin{align*}
\text{Source } s & \quad \text{Link } l \\
\text{Network} & \quad \text{Network}
\end{align*}
\]
Duality: Nonconvex Case

Application-Awareness cycles:
First wave: linearity (1940s-1950s)
Second wave: convexity (1980s-1990s)
Third wave: nonconvexity

- Relax polynomial positivity to sum-of-squares (SOS) decomposition
- By Positivstellensatz in real algebraic geometry, one can turn constrained polynomial minimization into polynomial-time SDP
- Nested family of SDP relaxations that works efficiently in practice

Network applications: Best multi-user detector and optimal rate allocation for inelastic traffic
Geometric Programming

Basic geometric inequality: \( \frac{a+b}{2} \geq \sqrt{ab} \)

Surprising applications across the layers:
- Information theory
- Code design
- Wireless transceiver design
- Congestion control and power control
- Queuing theory
Applications

Large-scale academic test-beds and labs

- **TCP FAST** (Low)
- **WAN in Lab** (Low)
- **FAST Copper** (Chiang)
- **Internet 0** (Collaboration with MIT)

Industrial adoption

- **AT&T Labs** (Calderbank)
- **SBC** (Chiang)
- **Flarion Technologies** (Calderbank and Chiang)
Open Issues

Three prioritized problems

- Topology and dynamics of heterogeneous TCP
- Geometry and inelastic application rate allocation
- Stochastic network utility maximization

Other important open issues:

- Transient behaviors for layering as optimization decomposition
- Non-zero duality gap for layering as optimization decomposition
- Distributed SOS
- Unification of GP and SOS